Math 2150 - Homework # 1

What is a differential equation?

1. For the following ODEs, determine (i) what the order is, and (ii) if it is linear or non-linear.

   (a) $y' = e^{2x} + \sin(x)$

   (b) $y'' + \sin(2x)y' + 2y = e^x$

   (c) $y''' + 3y'' + 4y' + 12y = x^2 + x - 1$

   (d) $2y'' + yx^3y' + x^2y = 10$

   (e) $x^2\dfrac{d^{10}y}{dx^{10}} - 5\dfrac{d^2y}{dx^2} + \sin(x)\dfrac{dy}{dx} - 2\sin(x)y = e^x$

   (f) $(y^2+1)\dfrac{dy}{dx} + e^y y = 2x$

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2. Consider the second order linear ODE

$$y'' - 4y = 0 \tag{1}$$

   on the interval $I = (-\infty, \infty)$.

   (a) Show that $f_1(x) = e^{2x}$ and $f_2(x) = e^{-2x}$ are both solutions to (1).

   (b) Show that $f(x) = c_1e^{2x} + c_2e^{-2x}$ is a solution to (1) where c_1 and c_2 are any constants.

   (c) Find constants c_1 and c_2 so that $f(x) = c_1e^{2x} + c_2e^{-2x}$ solves the initial-value problem

$$y'' - 4y = 0, \quad y'(0) = 0, \quad y(0) = 1$$

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3. Show that $\phi(x) = 2\sqrt{x} - \sqrt{x}\ln(x)$ is a solution to the initial value problem

$$4x^2y'' + y = 0, \quad y'(1) = 0, \quad y(1) = 2$$

   on the interval $I = (0, \infty)$.

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4. Show that $f(x) = \cos(2x)$ solves the initial-value problem

$$y''' + 3y'' + 4y' + 12y = 0, \quad y''(0) = -4, \quad y'(0) = 0, \quad y(0) = 1$$

on $I = (-\infty, \infty)$.
